Journal of Modern Technology & Engineering
Vol.3, No.1, 2018, pp.1-14

FUZZY ANALYSIS, A POSSIBLE SOLUTION FOR THE STANDARD ERROR IN SAMPLING?

Willem K.M. Brauers[1,2*], Alvydas Baležentis[3], Tomas Baležentis[4]

[1]University of Antwerp, Faculty of Applied Economics, Antwerpen, Belgium
[2]Vilnius Gediminas Technical University, Vilnius, Lithuania
[3]Mykolas Romeris University, Vilnius, Lithuania
[4]Vilnius University, Vilnius, Lithuania

Abstract. For instance the Belgian CIM, which is doing market analysis for all Belgian newspapers, magazines and cinema, arrives for some local newspapers at a standard error of 15 or a spread of 30%. This approach is scientific nonsense but accepted by the publishers of advertisement. On the other side the usual standard error for market research is 5%.
Is it possible to avoid this Spread by Sampling? Here Multi-Criteria Decision Analysis may help. The MULTIMOORA method will solve the problems of normalization and of importance, whereas Fuzzy MULTIMOORA may take care of the annoying spread when a portion of a statistical population, a sample, is examined instead of the corresponding totality.
An example of market research, namely the construction of dwellings in Lithuania, illustrates the theory.

Keywords: *MULTIMOORA, decision matrix, ordinal dominance, sample, standard error, spread.*

Corresponding Author: *Prof., Dr.Econ., Dr. hon. c., Willem K. M. Brauers, University of Antwerp, Faculty of Applied Economics, Prinsstraat, 13, Antwerpen, 2000 Belgium; Vilnius Gediminas Technical University, Sauletekio al., Vilnius 10221, Lithuania, e-mail: willem.brauers@uantwerpen.be*

Manuscript received: *02 February 2018*

1. Introduction

In order to clarify the way of thinking in this text a universal classification of events is necessary. The *Statistical Universe* represents the set of all events in the past, the present and the future. *Prospective Thinking* as a part of the Statistical Universe will consider the future by combining creative thinking, like in Futurology, but with a sense of reality. It will consult all Stakeholders i.e. everybody interested in an issue, mostly represented by delegates. The stakeholders will by preference not be involved in open discussions but rather in nominal techniques such as Delphi, Nominal Group Techniques and Scenario Writing (examples are given in Brauers, 2004).

The *Statistical Population* forms a partition of the Statistical Universe namely as a subset of some events. The subset can be complete or incomplete. An example of a complete subset is the *Census of Population*. For the census, after the tradition, the parents of Jesus had to move to their birth place in order to participate in the Roman census. Nowadays the census is taken at the permanent residence after the indications of the Belgian statistician Quetelet in 1830 (Académie Royale de Belgique, 1974). Most of the time however, a complete subset of the statistical population is not possible. Instead one as to be satisfied by an incomplete subset named a sample or a poll. A *Sample* can be drawn from a statistical population. For instance, from a population of four million households a random sample of 4,000 households is drawn. These households will keep record of their consumption for instance during one year. This subset of the population is considered significant for the whole population. A *Poll* represents a possible and significant estimation of a future subset of the statistical universe to which it belongs. The distance between the opinion of the whole population and the sample is measured by the *standard deviation* in one direction and by the *spread*, being the double of the standard deviation, in both directions.

If the publicity capacity of a newspaper, magazine, cinema or television would be announced by these media themselves the public, the publicity brokers in particular, would have no confidence in the outcome. Therefore a neutral institution will deliver the results by sampling. CIM is for instance the organization concerned in Belgium. Results for 2013-14 show as an average for all newspapers, magazines and cinema a standard deviation of 12 % with for some newspapers a standard deviation of 15% or a spread of 30%. The results are scientifically not acceptable but the publicity brokers prefer these results above eventual statistics from the newspapers themselves (CIM 2013-2014, CIM 2014).

Gallup Polls concern public opinion measurement, general elections in particular. For the 2016 Brexit election a leaving of Great Britain out the European Union was wrongly predicted as negative and in the same year the prediction for the election of a president of the United States was wrong too.

Up till now the mentioned applications were mono-objective, whereas next application is more general i.e.: Multi-Objective.

Some may think that the advent of **Big Data** will solve the problem of sampling. A department store may get all consumer habits of a customer and by extension of all its customers, but not of the non-customers perhaps a more interesting part for its expansion. It could make a deal with all other department stores but even then a part of the population is not entering any department store. Consequently, sampling remains necessary.

2. The example of Market Research

Market research mostly works with a confidence level of 95%, which means a 5% probability that outside conditions will interfere. On the other side for instance a dam against flooding has to have a confidence level of 99.9%, i.e. a probability of 1 on 1,000 that the dam will be too low or will collapse.

Also the size of the sample is important. Marketing accepts for instance 100 interviews with a standard error of:

$se = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.25}{100}} = 0.05$ which means 5% under or 5% above the real percentage (p = expected probability; q the opposite$q = 1 - p$).

In a normal distribution: $q = p = 0.5$. The sum of the 5% under plus the 5% above the real percentage or the sum of the standard errors is called the **Spread**. Hoel (Hoel, 1971) speaks of the extent of the spread, whereas Hays (Hays 1973) calls it spread or dispersion. Mueller et al. (Mueller *et al.*,1970) speak rather of “Range”.

An example is taken from: Brauers et al., 2008. Construction, taking off, maintenance and facilities management of a building are typical examples of consumer sovereignty: the new owner likes to have a reasonable price to pay, to have confidence in the contractor, to know about the duration of the works, the service after completion and the quality of the work. On the other side the contractor has his objectives too, like the satisfaction of the client, diminishing of external costs and annoyances and the management cost per person employed as low as possible. In other words it concerns a problem of multi-objectives. Therefore a final ranking will show the best performing contractor from the point of view of the clients but also from the point of view of the contractors.

The largest contractors of dwellings in Vilnius, the capital of Lithuania, were approached, of which 15 agreed to fix and estimate their main objectives, namely 9 objectives as given in Table 1.

Table 1. Main objectives of contractors of dwellings in Vilnius

1. Cost of building management Lt/m2 *min*
2. Cost of common assets management Lt/m2 *min*
3. HVAC system maintenance cost (mean) Lt/m2 *min*
4. Courtyard territory cleaning (in summer) Lt/m2 *min*
5. Total service cost Lt/m2 *min*
6. Length of time in maintenance business experience in years *max*
7. Market share for each contractor % *max*
8. Number of projects per executive units/person *max*
9. Evaluation of management cost (C_{min} / C_p) *max*

Table 2 summarizes the reaction of the contractors on the proposed objectives.

Table 2. Initial decision making matrix of 15 contractors of dwellings in Vilnius (a)

Objectives ↓

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Alternatives ↔	**MIN.**	**MIN.**	**MIN.**	**MIN.**	**MIN.**	**MAX.**	**MAX.**	**MAX.**	**MAX.**
a_1	0.064	0.11	0.18	0.31	0.67	12	11.75	4.6	0.83
a_2	0.06	0.14	0.37	0.12	0.5	3	0.39	0.33	0.885
a_3	0.057	0.11	0.18	0.15	0.69	12	5.25	1.47	0.935
a_4	0.06	0.12	0.10	0.15	0.57	12	7.1	2.78	0.9
a_5	0.058	0.1	0.18	0.2	0.45	12	5.56	1.39	0.9
a_6	0.071	0.3	0.18	0.26	0.82	13	26.62	5.67	0.746
a_7	0.11	0.14	0.18	0.12	0.55	5	2.82	1.2	0.483
a_8	0.058	0.18	0.37	0.19	0.61	11	9.48	3.03	0.916
a_9	0.053	0.14	0.16	0.23	0.8	11	2.23	0.8	1
a_{10}	0.07	0.26	0.29	0.2	0.7	11	13.5	9.05	0.75
a_{11}	0.12	0.2	0.09	0.2	0.81	4	4.7	1.5	0.443
a_{12}	0.071	0.28	0.18	0.28	0.73	12	2.35	0.86	0.746
a_{13}	0.078	0.2	0.18	0.3	0.76	8	5.6	3.25	0.681
a_{14}	0.056	0.14	0.18	0.12	0.5	11	2.66	1.7	0.948
a_{15}	0.12	0.14	0.09	0.21	0.56	3	0.04	0.03	0.531

(a) Brauers et al, 2008, 250.

From information of the Dwelling Owners Association, a panel of 30 owners of dwellings chosen at random agreed with these 9 objectives, but they increased the objectives with 11 other ones (These additional objectives were: standard of management services, maintenance of common property, work organization, effectiveness of information use, certification of company, range of services, reliability of company, company reputation, staff qualification and past experience, communication skills, geographical market restrictions.). However these additional objectives were only expressed in qualitative points showing some overlapping and after their rating represented only 25.9% importance of the total. If these opinions are only taken as indicative these qualitative objectives can be dropped.

For the 9 objectives with 30 interviews even chosen at random mean a confidence level of: standard error $se = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.25}{30}} = 0.09$, which means 9% under or 9% above the real percentage or a Spread of 18%.

Beside this formula: one has to be aware of the *Universe* or *Population* around the sample (Mueller *et al.,* 1970) which is not directly quantitative:

- only the Vilnius population above the age of 18 has to be taken into consideration and in addition only households;
- an advance payment for buying property of 15 to 30% is needed in Lithuania (Swedbank, 2012);
- only 13% of the Vilnius population is willing to take a mortgage (SEB, 2013,6). From this 13% has to be excluded: existing mortgages, buying an existing property, buying a social apartment or people not interested in the location in question;
- saving rate in Lithuania was only 1.92% in 2008, which is extremely low. In 2009 there was even dissaving (Statistics Lithuania, 2014).

Accepting the 18% spread for a limited universe one may conclude that the 30 respondents are representative for the potential buyers of the proposed property in Vilnius.

The nature of the construction industry involves that the total number of the minima is mostly larger than the total number of the maxima, which is the case here. Instead of attributing significance coefficients the contractors and the small sample of owners preferred the *Attribution of Sub-Objectives*. Indeed, five objectives on nine concern the super objective minimization of costs. Even, the last maximization forms in fact a cost consideration.

The topic of this research is to find a method in such a multi criteria problem of sampling in order to make a choice in a rational way, to come to an optimum for the results and to interpret them. The remark can be made that the examples of the Belgian newspapers and the Gallup polls are only mono-objective. However they could be considered as special cases of multi-objective optimization.

3. Search for a robust method to make a choice in a rational way between different solutions responding to different objectives

A **Decision Matrix**, assembles raw data with vertically numerous objectives, criteria (a weaker form of objectives) or indicators and horizontally alternative solutions, like projects.

SAW, followed by many other methods, such as the different ELECTRE variations (Roy since 1966, with many variations in Electre since then, see therefore Schärlig, 1985; 1996), PROMETHEE (Brans & Mareschal, 2005), AHP and ANP (Saaty since 1988), reads the decision matrix in a horizontal way. The Additive

Weighting Procedure (MacCrimmon, 1968; which was called SAW, Simple Additive Weighting Method, by Hwang and Yoon, 1981) starts from:

$$MaxU_j = w_1x_{1j} + w_2x_{2j} + \cdots + w_ix_{ij} + \cdots + w_nx_{nj}$$

U_j =overall utility of alternative j with $j = 1,2,\ldots\ldots,m,$ m the number of alternatives
w_i =weight of attribute i indicates as well as normalization as the level of importance of an objective with the condition

$$\sum_{i=1}^{n} w_i = 1,$$

$i = 1,2,\ldots\ldots,n;$ n the number of attributes or objectives,
x_{ij} =response of alternative j on attribute i.

As the weights add to one a new super-objective is created and consequently it becomes difficult to speak of multiple objectives.

With weights importance of objectives is mixed with normalization. Indeed weights are mixtures of normalization of different units and of importance coefficients.

Neither can be thought of methods comparing objectives or alternative solutions two by two with in this way being a victim of the Condorcet-Arrow Paradox (Condorcet, 1785; Arrow, 1963). Vertical reading of the Decision Matrix means that normalization is not needed as each column is expressed in the same unit. In addition if each column is translated in ratios dimensionless measures are created and the columns become comparable to each other. Indeed they are no more expressed in a unit. Different kind of ratios are possible, but Brauers and Zavadskas (2006) proved that the best one is based on the square root in the denominator.

$$x_{ij}{}^* = \frac{x_{ij}}{\sqrt{\sum_{j=i}^{m} x_{ij}^2}} \qquad (1)$$

Vertical reading of the decision matrix and the Brauers-Zavadskas ratios are practiced in the MOORA method:

$$y_j{}^* = \sum_{i=1}^{g} x_{ij^*} - \sum_{i=g+1}^{n} x_{ij^*} \qquad (2)$$

$i = 1,2,\ldots,g,$ objectives to maximized,
$i = g+1, g+2,\ldots,n$ objectives to minimized,
$y_j{}^*$= alternative j concerning all objectives and showing the final preference.

A second Method used in MOORA is the *Reference Point Approach* which will use the ratios found earlier but now linked to a Maximal Objective Reference Point. The Maximal Objective Reference Point approach is called realistic and non-subjective as the co-ordinates (r_i), which are selected for the reference point, are realized in one of the candidate alternatives. In the example: A(10;100), B(100;20) and C(50;50) the maximal objective reference point R_m results in: (100;100). Per objective the coordinates of the corresponding ratio are subtracted from the coordinates of the Reference Point.

Then these results are subject to the *Metric of Chebyshev* (Chebyshev, 1947; Karlin & Studden, 1966):

$$\min_{(j)}\left\{\max_{(i)}\sqrt{\left(r_i - x_{ij}{}^*\right)^2}\right\} \tag{3}$$

r_i = the i^{th} co-ordinate of the reference point,
$x_{ij}{}^*$= the dimensionless measurement of objective i for alternative j,
$i = 1,2\ldots\ldots n;\ n$ the number of objectives or attributes,
$j = 1,2\ldots\ldots m;\ m$ the number of alternatives.

Reference Point Methods like TOPSIS (Hwang & Yoon, 1981) and VIKOR (Opricovic & Tzeng, 2004) do not use weights but rather dimensionless measures but they are overtaken by MOORA which is composed of two different dimensionless based methods, each controlling each other.

An interesting example of MOORA compared with other methods is what Chakraborty has done for industrial management. Chakraborty (Chakraborty, 2011) checked the above mentioned famous methods of Multi-Objective Decision Making for decision making in manufacturing with MOORA, showing to be better for: computational time, simplicity, mathematical calculations, stability and information type.

Karuppanna and Sekar (Karuppanna & Sekar, 2016) studied the several approaches not only towards Manufacturing but also to the Service Sectors, with the same results, which is extremely important for the underlying study.

To the two methods of MOORA a third method is added: the Full Multiplicative Form. The use of three different methods of MOO is more robust than using of two, making MULTIMOORA superior to all existing methods of Multiple Objectives Optimization.

In The Full Multiplicative Form per row of an alternative all objectives are simply multiplied, but the objectives to be minimized are parts of the multiplication process as denominators.

$$U_j = \prod_{i=1}^{n} x_{ij} \tag{4}$$

with:
$j = 1, 2, \ldots, m;\ m$ the number of alternatives,
$i = 1, 2, \ldots, n;\ n$ being the number of objectives,
x_{ij} = response of alternative j on objective i,
U_j = overall utility of alternative j.

$$U_j' = \frac{A_j}{B_j}$$

$$A_j = \prod_{g=1}^{i} x_{gj}$$

where: $j = 1, 2, \ldots, m;\ m$ the number of alternatives,
i = the number of objectives to be maximized.

$$B_j = \prod_{k=i+1}^{n} x_{kj}$$

In MOORA a summary of the two methods was made on view, impossible for MULTIMOORA. Adding of ranks, ranks mean an ordinal scale (1st, 2nd, 3rd etc.) signifies a return to a cardinal operation (1 + 2 +3 + …). Is this allowed? The answer is "no" following the Noble prize Winner Arrow:

The Impossibility Theorem of Arrow

"Obviously, a cardinal utility implies an ordinal preference but not *vice versa"* (Arrow, 1974).

Axioms on Ordinal and Cardinal Scales

1. A deduction of an Ordinal Scale, a ranking, from cardinal data is always possible.
2. An Ordinal Scale can never produce a series of cardinal numbers.
3. An Ordinal Scale of a certain kind, a ranking, can be translated in an ordinal scale of another kind.

In application of axiom 3 the rankings of three methods of MULTIMOORA are translated into another ordinal scale based on *Dominance*, *being Dominated, Transitivity and Equability* (Ordinal Dominance Theory).

The whole process becomes clear in the following Fig.1, whereas Fuzzy MULTIMOORA will give the corresponding formulas of the three methods later on.

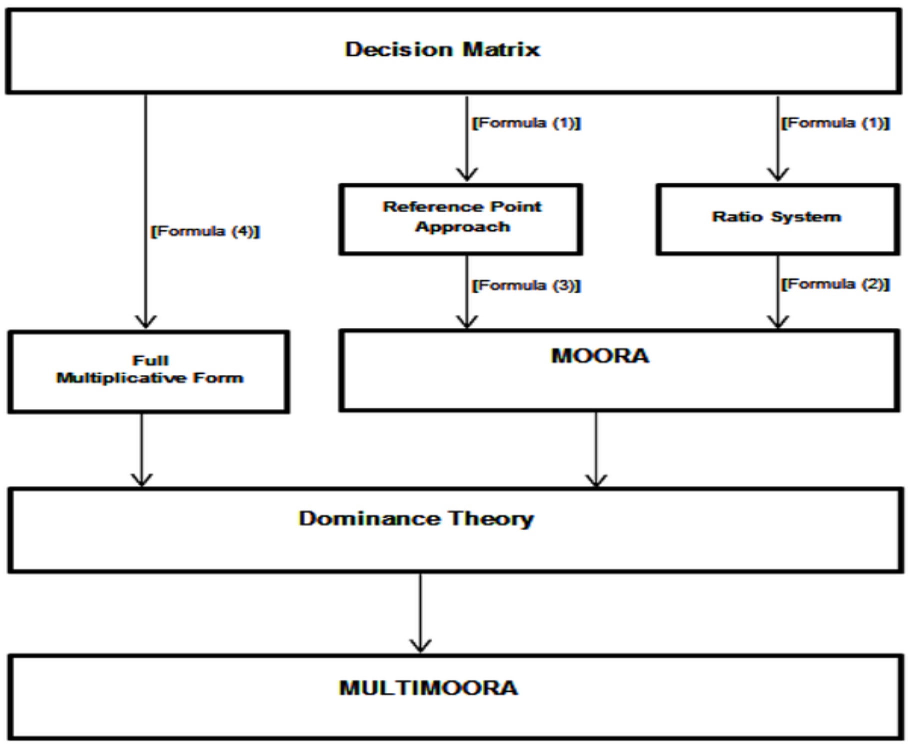

Fig 1. MULTIMOORA

The spread still remains in MULTIMOORA. Fuzzy MULTIMOORA will try to remove the spread by extending the numbers on both sides till the standard deviation as shown in next Table 3.

Table 3. Ranking Contractors with 9% less and 9% more for each objective

	Obj. 1		Obj. 2, 3, 5		Obj.	6	Obj.7, 8		Obj.	9
0.058	0.064	0.070	---------------------------	10.92	12	13.08	-------------	0.76	0.83	0.90
0.055	0.060	0.065	---------------				------------- -	0.81	0.89	0.96
0.052	0.057	0.062	---------------				------------- -	0.85	0.94	1.02
0.053	0.058	0.063	---------------				------------- -	0.83	0.91	0.99
0.053	0.058	0.063	---------------				------------- -	0.83	0.91	0.99
0.065	0.071	0.077	---------------				------------- -	0.68	0.75	0.81
0.100	0.110	0.120	---------------				------------- -	0.44	0.48	0.53
0.053	0.058	0.063	---------------				------------- -	0.83	0.92	1.00
0.048	0.053	0.058	---------------				------------- -	0.91	1.00	1.09
0.065	0.071	0.077	---------------				------------- -	0.68	0.75	0.81
0.109	0.120	0.131	---------------				------------- -	0.40	0.44	0.48
0.065	0.071	0.077	---------------				------------- -	0.68	0.75	0.81
0.071	0.078	0.085	---------------				------------- -	0.62	0.68	0.74
0.051	0.056	0.061	-------------				------------- -	0.86	0.95	1.03
0.109	0.120	0.131	---------------				------------- -	0.48	0.53	0.58

27 objectives and sub-objectives replace the 9 objectives

Consumer Sovereignty will play by giving to each objective a minus value or a max value of 9% deviation corresponding with the confidence level. For instance input of contractor a_1 into objective 6 being 12 is replaced by 10.92, 12 and 13.08.

In the given example, *Consumer's Attitude on Contractor's Ranking*, it is not certain that a contractor will accept the changes, proposed by the client, as it means a change in his offer.

In addition, the ranking of the contractors may change. Perhaps Fuzzy MULTIMOORA could bring the answer.

4. The Fuzzy MULTIMOORA Method

4.1. Fuzzy Numbers

Being a special case of the fuzzy sets, fuzzy numbers express uncertain quantities. Among various instances of fuzzy numbers, the triangular fuzzy numbers are often used for multi-criteria decision making. A triangular fuzzy number $\widetilde{x}$ can be represented by a tripet: $\tilde{x} = (a, b, c)$, where a and c are the minimum and maximum bounds, respectively, and b is the modal value or kernel (Kaufmann & Gupta, 1991).

The following arithmetic operations are available for the fuzzy numbers (Wang & Chang, 2007):

1. Addition $\oplus$:

$$\tilde{A} \oplus \tilde{B} = (a, b, c) \oplus (d, e, f) = (a + d, b + e, c + f); \qquad (5)$$

2. Subtraction! :

$$\tilde{A}!\tilde{B} = (a,b,c)!(d,e,f) = (a-f, b-e, c-d); \qquad (6)$$

3. Multiplication ⊗ :

$$\tilde{A} \otimes \tilde{B} = (a,b,c) \otimes (d,e,f) = (a \times d, b \times e, c \times f); \qquad (7)$$

4. Division %:

$$\tilde{A} \% \tilde{B} = (a,b,c) \% (d,e,f) = (a \setminus f, b \setminus e, c \setminus d). \qquad (8)$$

The vertex method can be applied to measure the distance between two fuzzy numbers. Let $\tilde{A} = (a,b,c)$ and $\tilde{B} = (d,e,f)$ be the two triangular fuzzy numbers. Then, the vertex method can be applied:

$$d(\tilde{A},\tilde{B}) = \sqrt{\frac{1}{3}[(a-d)^2 + (b-e)^2 + (c-f)^2]}.$$

4.2. Fuzzy MULTIMOORA Method

Fuzzy MULTIMOORA was introduced by Brauers et al. (Brauers *et al.*, 2011). In this study we employ the modified version as reported by Balezentiene et al. (Balezentiene *et al.*, 2013). The fuzzy MULTIMOORA begins with fuzzy decision matrix, $\tilde{X}$ where

$$\tilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3})$$

are aggregated responses of alternatives on objectives.

The Fuzzy Ratio System

The Ratio System defines normalization of the fuzzy numbers $\tilde{x}_{ij}$ resulting in matrix of dimensionless numbers. The normalization is performed by comparing appropriate values of fuzzy numbers

$$\tilde{x}_{ij}^* = (\tilde{x}_{ij1}^*, \tilde{x}_{ij2}^*, \tilde{x}_{ij3}^*) = \begin{cases} \tilde{x}_{ij1}^* = x_{ij1} \Big/ \sqrt{\frac{1}{3}\sum_{i=1}^{m}\left[(x_{ij1})^2 + (x_{ij2})^2 + (x_{ij3})^2\right]} \\ \tilde{x}_{ij2}^* = x_{ij2} \Big/ \sqrt{\frac{1}{3}\sum_{i=1}^{m}\left[(x_{ij1})^2 + (x_{ij2})^2 + (x_{ij3})^2\right]}, \qquad \forall i,j \\ \tilde{x}_{ij3}^* = x_{ij3} \Big/ \sqrt{\frac{1}{3}\sum_{i=1}^{m}\left[(x_{ij1})^2 + (x_{ij2})^2 + (x_{ij3})^2\right]} \end{cases} \qquad (10)$$

The normalization is followed by computation of the overall utility scores, $\tilde{y}_i^*$, for each i^{th} alternative. The normalized ratios are added or subtracted with respect to the type of criteria:

$$\tilde{y}_i^* = \sum_{j=1}^{g} \tilde{x}_{ij}^* ! \sum_{j=g+1}^{n} \tilde{x}_{ij}^* , \qquad (11)$$

where $g = 1,2,\ldots,n$ stands for number of criteria to be maximized. Then each ratio is $\tilde{y}_i^* = (y_{i1}^*, y_{i2}^*, y_{i3}^*)$ defuzzified:

$$BNP_i = \frac{y_{i1}^* + y_{i2}^* + y_{i3}^*}{3} \qquad (12)$$

BNP_i denotes the best non-fuzzy performance value of the i^{th} alternative.
Consequently, the alternatives with higher BNP values are attributed with higher ranks.

The Fuzzy Reference Point

The fuzzy Reference Point approach is based on the fuzzy Ratio System. The Maximal Objective Reference Point (vector) $\tilde{r}$ is found according to ratios found in Eq. 10. The j^{th}coordinate of the reference point resembles the fuzzy maximum or minimum of the j^{th} criterion, $\tilde{x}_j^+$ where

$$\begin{cases} \tilde{x}_j^+ = \left(\min_i x_{ij1}^*, \max_i x_{ij2}^*, \max_i x_{ij3}^*\right), & j \leq g; \\ \tilde{x}_j^+ = \left(\min_i x_{ij1}^*, \min_i x_{ij2}^*, \min_i x_{ij3}^*\right), & j > g. \end{cases} \tag{13}$$

Then the every element of normalized responses matrix is recalculated and final rank is given according to deviation from the reference point (13) and the Min-Max Metric of Chebyshev:

$$\min_i \left(\max_j d\left(\tilde{r}_j, \tilde{x}_{ij}^*\right)\right). \tag{14}$$

The Fuzzy Full Multiplicative Form

Overall utility of the i^{th} alternative can be expressed as a dimensionless number by employing (8)

$$\widetilde{U}_i^1 = \tilde{A}_i \% \tilde{B}_i \;, \tag{15}$$

$$\tilde{A}_i = (A_{i1}, A_{i2}, A_{i3}) = \prod_{j=1}^{g} \tilde{x}_{ij} \;, \qquad i = 1,2,\dots,m$$

denotes the product of objectives of the i^{th} alternative to be maximized with $g = 1, \dots, n$ being the number of criteria to be maximized.

$$\tilde{B}_i = (B_{i1}, B_{i2}, B_{i3}) = \prod_{j=g+1}^{n} \tilde{x}_{\ddot{y}}$$

denotes the product of objectives of the i^{th} alternative to be minimized with$n$ - g as the number of criteria to be minimized. Since the overall utility $\widetilde{U}_i'$ is a fuzzy number, one needs to defuzzify it to rank the alternatives (12). The higher the best non-fuzzy performance value (BNP), the higher will be the rank of a certain alternative.

Thus, the fuzzy MULTIMOORA summarizes fuzzy MOORA (i.e. fuzzy Ratio System and fuzzy Reference Point) and the fuzzy Full Multiplicative Form.

The case: *Consumer's Attitude on Contractor's Ranking* employs then this method while, as said before, to each objective a min or a max value of 9%, corresponding with the confidence level, is given. For instance input of contractor a_1 into criterion 6 being 12 is replaced in a fuzzy reasoning by 10.92, 12 and 13.08 (see Table 3). A voter can give more importance to contractor a_1 and to criterion 6 by preferring 13.08 above 12. There is even more: each point between 10.92 and 13.08 is possible. In taking rows and columns in Table 3 the numbers will have more or less the form of an upside down Gauss Curve, however not standard normal or symmetrical (Hoel, 1971) but skewed (Hays, 1973) and with the restriction that the solutions are not continuous but discrete. Fuzzy means also that all points on a line linking all values of each alternative solution, here a contractor, are also possible. Nevertheless it is sufficient only to take into account the extreme positions, in the given example (Table 3) 10.92 and 13.08.

The three parts of Fuzzy MULTIMOORA present the following results in Table 4. The summary of the three parts is made by the Ordinal Dominance Theory as explained earlier.

Table 4. Ranking by Fuzzy MULTIMOORA after its three parts and with the application of Ordinal Dominance Theory (a)

	Fuzzy Ratio System	Fuzzy Reverence Point Method	Fuzzy Multiplicative Form	Fuzzy MULTIMOORA
a6	1	1	3	1
a1	2	3	2	2
a10	3	2	4	3
a4	4	5	1	4
a5	5	7	5	5
a3	6	8	6	6
a8	8	4	7	7
a14	7	11	8	8
a13	10	6	9	9
a9	9	13	10	10
a7	12	10	11	11
a11	13	9	12	12
a12	11	12	13	13
a2	14	14	14	14
a15	15	15	15	15

(a) Calculations available from the authors

Table 5 ranks the three possibilities for refining the studied market research.

Table 5. Ranking Contractors after the three Possibilities (a)

MOORA with 18% spread		MULTIMOORA with 18% spread		Fuzzy MULTIMOORA no spread	
a6	1	**a6**	**1**	**a6**	**1**
a10	2	**a4**	**2**	**a1**	**2**
a1	3	a10	3	a10	3
a4	4	**a1**	**4**	**a4**	**4**
		a5	5	a5	5
		a3	6	a3	6
		a8	7	a8	7
		a14	8	a14	8
		a13	9	a13	9
		a9	10	a9	10
		a7	11	a7	11
		a11	12	a11	12
		a12	13	a12	13
		a2	14	a2	14
		a15	15	a15	15

(a) Calculations available from the authors. To make it easier to understand MULTIMOORA or in particular to apply MULTIMOORA for marketing research the software is made in Excel style: first in numbers and then in control modus for formulas. For Excel applications, see: Herkenhoff and Fogli, 2013; Quirk, 2011.

Contractor a_6 is preferred overall, which brings even more certainty on this solution. Contrary to MULTIMOORA with 18% spread Contractor a_1 is the second best as the method without spread shows its domination on the remaining other ones.

Nevertheless one has to be aware about the real outcome. In the worst case it could be that a client asks for a 9% additional effort from the side of the contractor. Can the winning contractor not anticipate this situation? Of course he can, however with the danger that the winning contractor would become one of his colleagues.

On the other side the contractor will be quasi certain that the client will buy his constructions, unless outside influences would interfere.

The theory is of general use in Multi-Objective Optimization each time a sample replaces total data mining around a certain phenomenon.

In the case of the Belgian media, being mono-objective, diminishing the standard deviation is the only possibility.

Regarding Gallup Polls concerning public opinion, general elections in particular, some additional research would be welcome.

5. Conclusion

The Belgian society called CIM is doing marketing research for all Belgian newspapers, magazines and cinema arriving at a spread of 24% as an average for all newspapers and even for some local newspapers at a spread of 30%, which is scientific nonsense but accepted by the publishers of advertisement. On the other side technical problems will ask for a much smaller standard deviation like for instance a standard error of 0.1% for the possibility that a dike is not strong enough for an eventual spring tide. Something in between the usual standard error for marketing research accepted is 5%.

Is it possible to avoid this Spread by Sampling? Here Multi-Objective Optimization Methods may be helpful with the additional question: which methods of MOO are useful in this case. It could not be methods based on the SAW principle as the choice of weights is another point of uncertainty. Neither can be thought of methods comparing objectives or alternative solutions two by two with in this way being a victim of the Condorcet-Arrow Paradox. Rather preference has to be given to methods based on dimensionless measurements like in the MOORA and MULTIMOORA Methods.

To the Ratio Method and the Reference Point Method of MOORA a third method is added in MULTIMOORA: the Full Multiplicative Form. The use of three different methods of MOO is more robust than using one or two.

Decision Making can be quantified by setting up a Decision Matrix with for instance Objectives or Criteria as columns and alternative solutions like Projects as rows. In this study Decision Making is quantified in its objectives, with the problem of normalization, due to the different units of the objectives and with the problem of importance. A MULTIMOORA method, chosen for its robustness instead of many other competing methods, will solve the problems of normalization, whereas Fuzzy MULTIMOORA will take care of the annoying spread in the samples.

Beside this method one has to be aware of the Universe around the sample, which is not directly quantitative. The Universe has not to be a disturbing factor.

It was Fuzzy MULTIMOORA which brought the solution to the Spread Problem by considering all the possible extreme positions delivered by the standard error.

The example of disclosing the desiderata of potential buyers of property in Lithuania, being Multi-Objective, presents an illustration of the theory. In the mono-objective study concerning the Belgian Media the diminishing of the standard error remains the only possibility. For Gallup polls concerning elections some additional research would be welcome.

Acknowledgements. We are grateful to the co-authors: Zavadskas, Turskis & Vilutiene for the basic data provided in the *Journal of Business Economics and Management*, 2008.

Competing Interests and Funding. There are no Competing Interests and there was no Funding.

Authors' Contributions. The study is a result of a teamwork between the three authors.

References

Académie Royale de Belgique (1974). *Adolphe Quetelet, 1796-1874*, Ministère des Affaires Economiques, Brussels.

Arrow, K.J. (1974). General Economic Equilibrium: Purpose, Analytic Techniques, Collective Choice, *American Economic Review*, 256.

Arrow, K.J. (1963). *Social Choice and Individual Values*, 2nd edit. Yale University Press, New Haven, 93 (First ed. Wiley, New York, 1951).

Balezentiene, L., Streimikiene, D., & Balezentis, T. (2013). Fuzzy decision support methodology for sustainable energy crop selection. *Renewable and Sustainable Energy Reviews*, *17*, 83-93.

Brans, J.P., & Mareschal, B. (2005). PROMETHEE methods. In *Multiple criteria decision analysis: state of the art surveys* (pp. 163-186). Springer, New York, NY.

Brauers, W.K.M. (2004). *Optimization Methods for a Stakeholder Society. A Revolution in Economic Thinking by Multi-objective Optimization*. Kluwer Academic Publishers and Springer, Boston.

Brauers, W.K.M., Zavadskas, E.K. (2006).The MOORA method and its application to privatization in a transition economy. *Control and Cybernetics*, *35*(2), 445-469.

Brauers, W.K.M., Baležentis A.& Baležentis T. (2011). MULTIMOORA for the EU Member States updated with fuzzy number theory. *Technological and Economic Development of Economy*, *17*, 273–304.

Brauers, W.K.M., Zavadskas, E.K., Turskis, Z.& Vilutiene, T. (2008). Multi-Objective Contractor's Ranking by applying the MOORA Method. *Journal of Business Economics and Management*, *9*(4), 245-255.

Chakraborty, S. (2011). Applications of the MOORA method for decision making in manufacturing environment. *International Journal Advanced Manufacturing Technology*, *54*, 1155-1166, Springer, DOI 10.1007/s00170-010-2972-0.

Chebyshev, P.L.(1947). *Complete Collected Works*, Moscow, Leningrad.

CIM September (2014). *2013-2014/1.Tactische Studie Pers & Bioscoop, September 2014, Méthodologie* (Tactical Study Press and Cinema, Methodology) www.cim.be/media/pers/bereik/methodologie.

CIM 2013-2014. *Year Report 2013-2014*, www.cim.be/media/pers/bereik/resultaten.

Condorcet, Marquis de (1785). *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix*, Paris, l'Imprimerie royale.

Hays, W.L. (1973). *Statistics for the Social Sciences*, second edition, Holt et al., London.

Herkenhoff, L., Fogli, J. (2013). *Applied Statistics for Business and Management using Microsoft Excel*, Springer, New York.

Hoel P.G. (1971). *Elementary Statistics*, Wiley, New York.

Hwang, C-L., Kwansun, Yoon. (1981). Multiple Attribute Decision Making, Methods and Applications. *Lecture Notes in Economics and Mathematical Systems*, *186*, Berlin, Springer.

Kaufmann, A., Gupta, M.M. (1991). *Introduction to Fuzzy Arithmetic: Theory and Applications*, New York, Van Nostrand Reinhold.

Karlin, S., Studden W.J. (1966). *Tchebycheff Systems: with Applications in Analysis and Statistics*. Interscience Publishers, New York.

Karuppanna, P.N., Sekar, K. (2016). Selection of Best Alternative in Manufacturing and Service Sector using Multigrade Decision Approach – a Review, *Computer science and Information Technology*, 57-74.

MacCrimmon & Kenneth R. (1968). *Decision Making among Multiple Attribute Alternatives. A Survey and Consolidated Approach*, RM-4823- ARPA, the Rand Corporation, Santa Monica (CAL).

Mueller, J.H., Schuessler K.F. & Costner, H.L. (1970). *Statistical Reasoning in Sociology*, second edition, Houghton Mifflin, Boston.

Opricovic S. & Tzeng G-H. (2004). Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS, *EJOR*, *156*, 445-455.

Roy, B., Benayoun, R. & Sussman B. (1966). *ELECTRE*, Société d'Economie et de Mathématique appliquées, Paris.

Quirk, T. (2011). *Excel 2010 for Business Statistics*, Springer, New York.

Saaty,T.L. (1988). *The Analytic Hierarchy Process*. Mcgraw-Hill, New York.

Schärlig, A., (1996). *Pratiquer Electre et Prométhée*, Presses polytechniques et universitaires romandes, Lausanne.

Schärlig, A., (1985). *Décider sur plusieurs critères*, Presses polytechniques romandes, Lausanne.

SEB, (2013). "Mortgage Market: where do intentions and behavior of the inhabitants differ?" (translated from Lithuanian) https://www.seb.lt/sites/default/files/web/pdf/SEB_prezentacija_paskolos1023.

Statistics Lithuania, (2014). Official Statistics Portal. National Accounts. http://osp.stat.gov.lt/en/rodikliai38.

Swedbank, (2012). Analysis of the real estate market: Inhabitants behavior and housing affordability" (translated from Lithuanian) http://www.swedbank.lt/lt/articles/view/1614.

Wang, T. C., Chang, T. H. (2007). Application of TOPSIS in evaluating initial training aircraft under a fuzzy environment. *Expert Systems with Applications*, *33*(4), 870-880.